STAT612 Project 3 (Revision 4)

Simulation of Compound Poisson Process

Dr. Muhammad Al-Hashimi

May 12, 2005

Due: Tuesday May 31, 2005

Let $\{X_{t_i} - X_{t_{i-1}} \,|\, i = 1, 2, \ldots\}$ denote exponentially-distributed times between arrivals with parameter λ, where $t_1, t_2, \ldots$ are the times of arrivals (1st arrival time, 2nd, and so on), and $t_0 = 0$. Let $Y_{t_1}, Y_{t_2}, \ldots$ be independent, uniformly distributed random variables each associated with their respective arrival event, and independent of arrival times, where $Y_{t_i} \in \{1, 2, \ldots, m\}$.

Write a program (programming environment of your choice) to simulate the *compound Poisson process* $Z = \{Z_t \,|\, t \geq 0\}$ obtained from summing $\{Y_{t_i} \,|\, t_i \leq t\}$, that is, all $Y_{t_i} \in (0, t]$.

Input

1. The arrival process rate λ.
2. The compound process jump maximum $m \geq 1$.
3. Number of sample path (function) simulation repetitions, $r > 0$.

Simulation Generate random variates from an exponentially-distributed sample with parameter λ to represent the inter arrival times $\{X_{t_i} - X_{t_{i-1}}\}$. At each event time t_i increment the process state by Y_{t_i}. The sample path simulation can be controlled by 3 methods (*suggestion*: use method 2[1]):

1. Predefined simulation time is reached.
2. Predefined number of arrivals is reached (for example: 1000 arrivals).
3. Adequate convergence of λ is obtained.

Repeat the sample path simulation r times. For sample path behavior study $r = 1$, and for ensemble behavior study $r > 1$.

Output For each sample path simulation compute:

1. The sequence of visited states Z_{t_i} and visiting times t_i.
2. The arrival rate for that sample.

Assignment Use the textbook as your reference for definitions and notation. Whenever possible, compare your results with theory from the textbook.

1. (Ensemble behavior) When $Y_{t_1} = Y_{t_2} = \cdots = 1$ (set $m = 1$) the compound process reduces to the familiar Poisson process.
   (a) Compute the ensemble average for each Z_t.
   (b) Use the simulation to determine if the process is strict sense stationary.
   (c) Use the simulation to determine if the process is time homogeneous.
   (d) Verify the mean arrival rate λ.
2. (Sample path behavior) For at least two values of λ, study 3 sample paths of the compound process:

[1] The arrival rate convergence method is the best. You may use a predefined time or number of arrivals for simplicity but be aware of the need to get a large enough sample to credibly approximate the desired random behavior.

(a) Plot the sample path (states vs. visiting times).

(b) We know from theory that the simple Poisson process is Markovian, time homogeneous counting process. Study the underlying DTMC by ignoring actual arrival times and observing the arrival sequence. Determine the transition matrix P of the DTMC.

(c) Use Maple to compute P^n of the DTMC. Discuss the resulting limiting behaior, if any.

Hints

1. You can better understand Z if you visualize it. Trace a few realizations of the process. Assume some arrival pattern on the time axis and associate a random value in $\{1, 2, \ldots, m\}$ with each arrival for some m. See how the process could unfold. Refer to Section 6.1 for examples.

2. Review Section 3.5 carefully to learn how to generate a random variate from an exponentially distributed sample. Pay special attention to Figure 3.25 and Example 3.12.

3. Two things could make or break your simulation work: experimentation and summary presentation of findings. Generally, most of your time should be spent on carefully designing your experiment, and on thinking about what results to show and how to show them. A big part of the experiment is to: (a) select your test cases to expose interesting behavior, and (b) ensure that your results are credible. The summary should allow the reader to clearly see your findings without having to go through volumes of raw data. Check the guidelines in the website for more details.

4. Be brief (without losing clarity). Really, this time. I will not take points but this will count as a minus when bonus points are awarded for this project.

5. More hints to come...