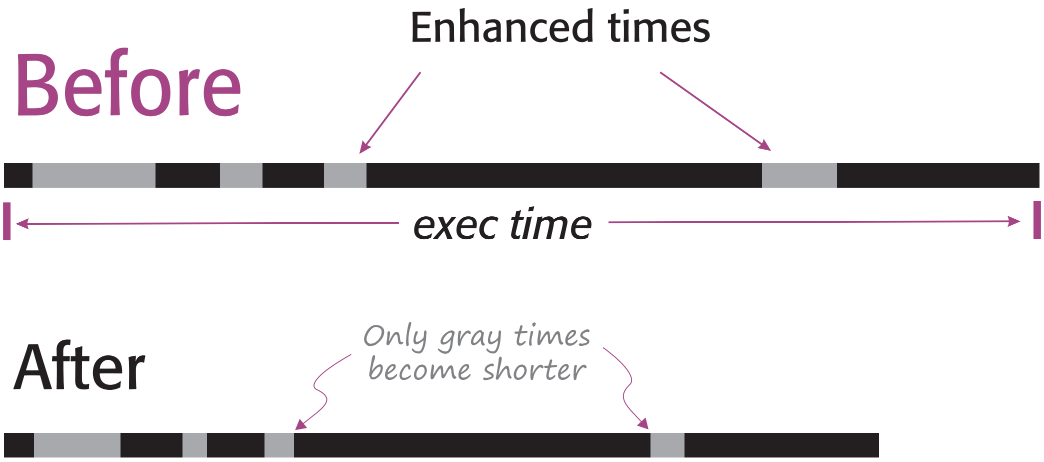
Enhancing A Part

Parts of a program are enhanced, affected times marked gray.

How much improvement in the execution time should be expected?

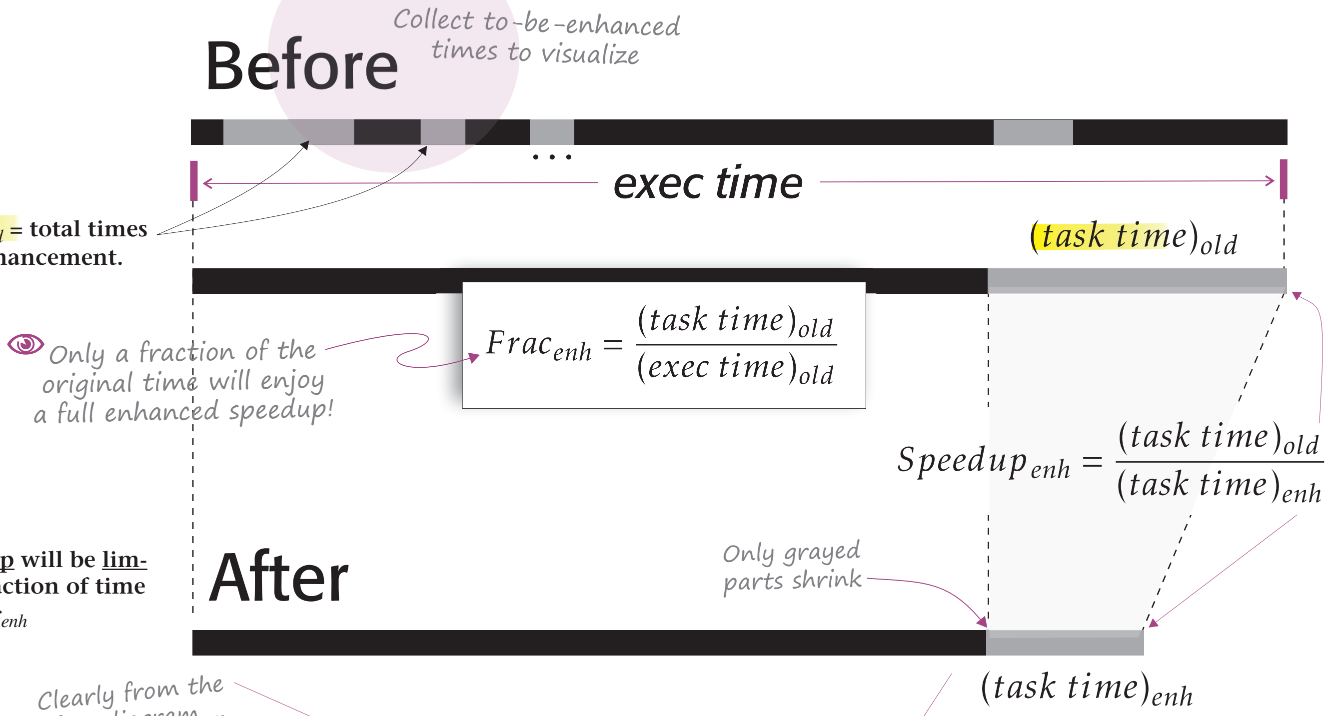


Conventionally, put bigger number in numerator to get non-fractional (>1) valued speedup ratio.

$$Speedup_{overall} = \frac{(exec\ time)_{old}}{(exec\ time)_{new}} = \ ?$$

Amdahl's Insight

Let $(task\ time)_{old}$ = total times affected by enhancement.



Overall speedup will be limited by that fraction of time enhanced, $Frac_{enh}$

Clearly from the after diagram ...

Fairly obvious relation leads to a speedup formula after a division and a couple of strategic substitutions (next).

$$(exec\ time)_{new} = \overbrace{(exec\ time)_{old} - (task\ time)_{old}}^{\text{same unaffected time}} + (task\ time)_{enh}$$

Amdahl's Law
A Formula

$$(exec\ time)_{new} = (exec\ time)_{old} - (task\ time)_{old} + (task\ time)_{enh}$$

1 2

$$\div\ (exec\ time)_{old}$$

To obtain an overall speedup term in the left-hand side

(in inverted form, flip back at the end)

and note

$$\boxed{\frac{(task\ time)_{old}}{(exec\ time)_{old}} = Frac_{enh}}$$

1

also observe (from previous slide)

another handy substitution for 3rd right-hand term after division

$$\frac{Frac_{enh}}{Speedup_{enh}} = \frac{(task\ time)_{old}}{(exec\ time)_{old}} \times \frac{(task\ time)_{enh}}{(task\ time)_{old}}$$

2

Exercise
Complete the derivation leading to the formula (next). *Ans. last slide.*

Amdahl's Law

For example, can't add 2nd processor and expect two-fold improvement in run time unless new processor is usable 100% of that time (i.e., $Frac_{enh} = 1$, which leads to overall speedup of 2).

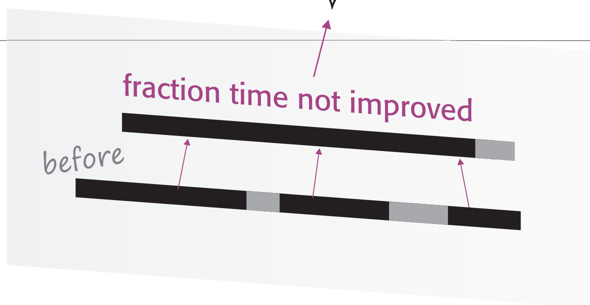
Quiz
Evaluate the limit as $Speedup_{enh} \to \infty$. Interpret results.

The limit clearly reveals that the fraction time not enhanced places an upper limit on overall speedup.

Exercise
Plot the speedup, overall vs. enhanced (e.g., 2 to 200 times) for different values of $Frac_{enh}$ (e.g., 0.9, 0.8, ..., 0.4). Comment on the shapes of the curves.

$$\frac{(exec\ time)_{old}}{(exec\ time)_{new}}$$

$$Speedup_{overall} = \frac{1}{\underbrace{(1 - Frac_{enh})}_{\text{fraction time not improved}} + \dfrac{Frac_{enh}}{Speedup_{enh}}}$$



Example 1 (CA:AQA)

Solution Hint. Figure out formula variables from problem wording, sometimes you need a small calculation.

Suppose that we are considering an enhancement that runs 10 times faster than the original machine but is only usable 40% of the time. What is the overall speedup gained by incorporating the enhancement?

Example 2 (CA:AQA)

Suppose a cache is 10 times faster than main memory, and suppose that the cache can be used 90% of the time. How much speedup do we gain by using the cache?

Example 3

Insight builder alert..
inspect the formula
carefully first

Suppose a program runs in 100 seconds on a computer, with multiply operations responsible for 80 seconds of this time. How much would multiplication need to be improved to make program run five times faster?

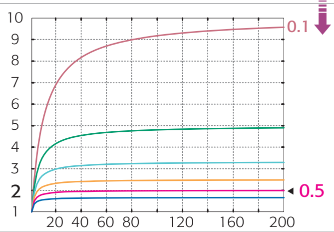
References & Answers

Exercise
(Graduate) Check the derivation of Amdahl's law in the original 1967 paper (!)

- Gene M. Amdahl, *Validity of the single processor approach to achieving large scale computing capabilities*, Proceeding AFIPS '67 (Spring) April 18-20, 1967, Spring Joint Computer Conference, pages 483-485.
- [CA:AQA] Hennessy and Patterson, *Computer Architecture: A Quantitative Approach*, Morgan-Kaufmann (ed. 2).

$1 - Frac_{enh}$

Each curve shows the diminishing returns featured by the formula. Also, overall speedup ceiling goes down as percent of time <u>unaffected</u> (un-enhanced) increases (note 2nd scenario for two-fold overall speedup).



$$\frac{(exec\ time)_{new}}{(exec\ time)_{old}} = 1 - \overbrace{\frac{(task\ time)_{old}}{(exec\ time)_{old}}}^{Frac_{enh}} + \overbrace{\frac{(task\ time)_{enh}}{(exec\ time)_{old}}}^{\frac{Frac_{enh}}{Speedup_{enh}}}$$

$$Speedup_{overall} = \frac{(exec\ time)_{old}}{(exec\ time)_{new}} = \frac{1}{(1 - Frac_{enh}) + \frac{Frac_{enh}}{Speedup_{enh}}}$$