Additional Useful Summation Results

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$$1 + 3 + 5 + 7 + \cdots + (2n-1) = n^2 \tag{1}$$

$$\sum_{i=m}^{n} i = m + (m+1) + \cdots + n = \frac{(n+m)(n-m+1)}{2} \tag{2}$$

$$\sum_{i=1}^{n} i^2 = 1 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \tag{3}$$

$$\sum_{i=0}^{n} 3^i = 1 + 3 + 9 + 27 + 81 + \cdots + 3^n = \frac{3^{n+1}-1}{2} \tag{4}$$

$$\sum_{i=0}^{n-1} a^i = 1 + a + a^2 + a^3 + \cdots + a^{n-1} = \frac{1-a^n}{1-a} \tag{5}$$

$$\sum_{i=0}^{n-1} i a^i = \frac{a - na^n + (n-1)a^{n+1}}{(1-a)^2} \qquad a \neq 1 \tag{6}$$

$$\sum_{i=0}^{\infty} \frac{1}{2^i} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = 2 \tag{7}$$

$$\sum_{i=0}^{k-1} \frac{1}{2^i} = 1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^{k-1}} = 2 - \frac{1}{2^{k-1}} \tag{8}$$

$$\sum_{i=0}^{n} \binom{n}{i} = 2^n \tag{9}$$

$$\sum_{i=1}^{n} i \log i \in \Theta(n^2 \log n) \tag{10}$$