General Plan Review

❶ Select suitable input size parameter *n*

❷ Identify a suitable basic operation

❸ Check basic operation count dependancy

❹ Setup a sum or a recurrence for $C(n)$

❺ Determine order of growth of $C(n)$
(may need to solve sum or recurrence)

Analysis of Recursive Algorithms
Simple Examples

Quiz
What could be another suitable input size parameter beside the magnitude of *n*? **Hint:** see textbook.

Exercise
Discuss possible basic operation choices for both cases, why would the multiplication be preferred for *Factorial*?

Algorithm $Factorial$
Input Integer $n \geq 0$
Output $n!$

if $n = 0$ **then**
    return 1
else **return** $Factorial(n - 1) \times n$

?

Exercise
Write a suitable efficiency recurrence in each case and solve using **backward substitutions**.

Algorithm $BinRec$
Input Integer $n > 0$
Output Number of bits in n's binary representation

if $n = 1$ **then**
    return 1
else **return** $BinRec(\lfloor n/2 \rfloor) + 1$

Solving Recurrences
Standard Solutions

Appendix B

- **Decrease-by-one**

$$T(n) = T(n-1) + f(n)$$

- **Decrease-by-constant-factor**

$$T(n) = \boldsymbol{T(n/b)} + f(n)$$

$$b > 1, n = b^k, k = 0, 1, 2, \cdots$$

- **Apply to simple examples**

cs223fig7.cdr
Saturday, February 5, 2022 2:18:01 PM
Color profile: Disabled

Composite Default screen

Tower of Hanoi

Quiz
What's a suitable input size parameter? What is the basic operation?

Quiz
How many moves solve pictured instance? *M*(?) = ? Hint: see Slide 8.

Easy to show (trivially)

$$n = 0 \quad \Rightarrow \quad M(0) = 0$$
$$n = 1 \quad \Rightarrow \quad M(1) = 1$$

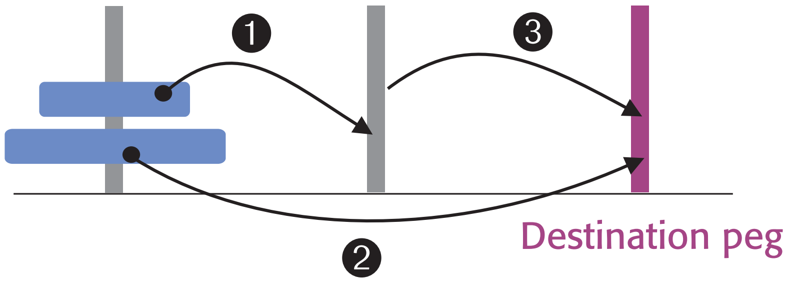
$$n = 2 \quad \Rightarrow \quad M(2) = 3$$ **why? next...**

cs223fig7.cdr
Saturday, February 5, 2022 2:18:01 PM
Color profile: Disabled

Composite Default screen

Tower of Hanoi
Case n=2

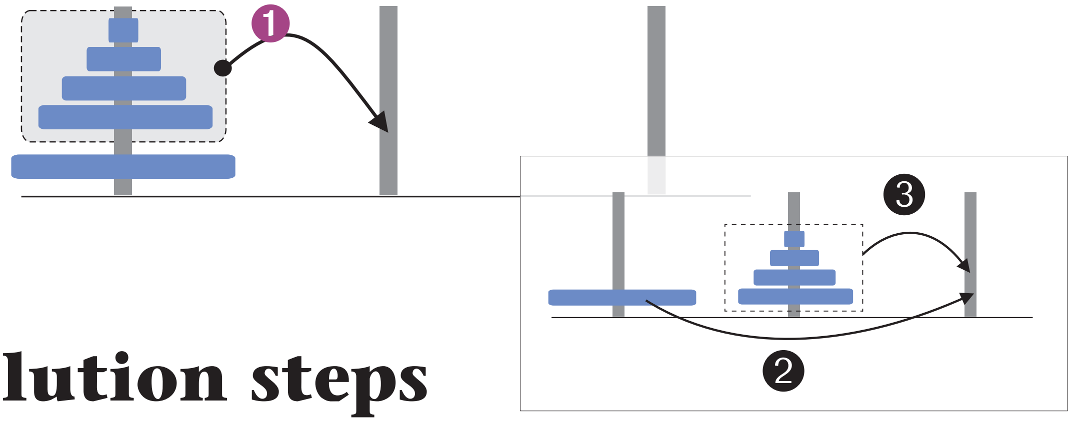
Exercise
Solve n=3 case by hand. How many steps were needed? **Hint:** use n=2 solution to move the top 2 disks to middle peg. Try a smartphone app.



Solution steps

❶ Top disk to middle peg
❷ Bottom disk to destination peg
❸ Top disk to destination peg

Tower of Hanoi
Generalized to n>2



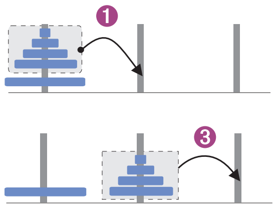
Quiz
How can we arrange top disks on middle peg (step 1)?

Solution steps

1. Move top <u>disks</u>
2. Move bottom disk to dest. peg
3. Move top <u>disks</u> to dest. peg

Use same procedure for instance n−1. If answer seems strange think of instance n=3. We already know how to solve n=2 case in 3 steps.

Tower of Hanoi
Observations



Steps 1 and 3

- ✎ Use same procedure to solve $n-1$
- ✎ Take same number of moves

Therefore, for steps 1 and 3

The number of moves is the same as was used to solve the $n-1$ case, $M(n-1)$

Tower of Hanoi
Solution Efficiency

Exercise
Solve the recurrence by backward substitutions.

Quiz
What's the sequence that satisfies the recurrence? **Hint**: use forward substitution, or ask *WolframAlpha*.

➪ In general, for n>1

step 1 step 2 step 3

$$M(n) = M(n-1) + 1 + M(n-1)$$
$$= 2M(n-1) + 1$$

➪ Check formula for n=1

Exercise
A fast robot can do 1000 moves/second (1 microsecond each). How long would it take to solve *n*=64? Ans. in session summary.

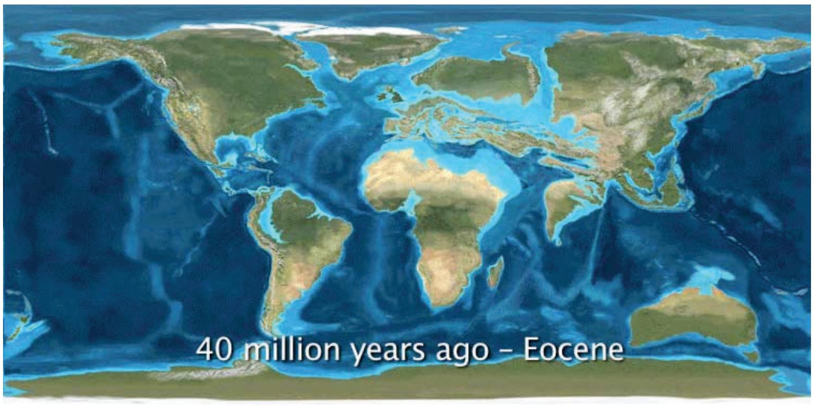
➪ How good? Solve recurrence!

Understand Exponential Growth
Visualization Exercise

Exponential Run Time Visualization			
Input Size	Basic Op Count	Run Time (s)*	Run Time (Yr)
10	1024	0	0
25	33554432	0	0
50	1,125,899,906,842,620	1	0
75	37778931862957200000000	37778932	1
100	1267650600228230000000000000000	1267650600228230	40,170,303

*Assuming a computer that can sustain 1 **petaflops** (10^{15} floating-point operations per sec), 1 move = average-flop

According to plate tectonics theory, 40 million years ago India had not yet collided (fully perhaps) with Asia!
https://en.wikipedia.org/wiki/Plate_tectonics



Quiz
What if a top (as of 2020) 100 peta-flop supercomputer was used?