Exercise
Write the basic operation count $C(n)$ for $n = 1, 2, 3, 4, 5$ as terms of a math sequence.

Efficiency from a sequence?

Algorithm $MaxElement$
Input Array $A[0..n-1]$
Output Largest value in A

$maxval \leftarrow A[0]$
for $i \leftarrow 1$ **to** $n-1$ **do**
  if $A[i] > maxval$ **then**
    $maxval \leftarrow A[i]$
return $maxval$

Do P5, P7

Another Useful Tool

- Mathematical sequence
- Generic (nth) term

Exercises
Lookup a formal definition of **sequences**, cite source (no *Mickey Mouse* sources please).

Examples

$$0, 1, 1, 2, 3, 5, 8, 13, \ldots \text{ (Fibonacci)}$$

0 1 2 3 4 5 6 7

Position of **term** is indicated by an index.

Quiz
What's the difference between a set and a sequence?

$$x(n) = 2n, n > 0 \qquad 2, 4, 6, 8, \ldots$$

Characterization

3 ways to do the same thing; each leads to the other two, but maybe more convenient or helpful in some cases.

Explicit sequence

0	1	3	6	10	15	21	…
0	1	2	3	4	5	6	

Generic term

$$x(n) = n(n+1)/2, \quad n \geq 1$$

Exercise
Use the **recurrence** to find the 8th term of the sequence. Verfiy from gen term. (*Note tricky position! Now on, we specify a position number directly.*)

Recurrence

$$x(n) = x(n-1) + n \text{ for } n > 0, \quad x(0) = 0$$

Recurrence Relations

Appendix B

- General solution
- Particular solution

Recurrence relations are used in analysis of recursive algorithms.

Definitions, examples

$$x(n) = x(n-1) + n \text{ for } n > 0, \quad x(0) = 0$$

$$x(n) = x(n/2) + n \text{ for } n > 1, \quad x(1) = 1$$

The condition on the generic term (previous slide) specifies a different sequence (correct it!).

Recurrence solution

Methods to solve (later)

Standard Recurrences

Appendix B

Quiz
Identify the terms related to the **generic term** in each recurrence (no math, use words).

➪ Decrease-by-one

$$T(n) = T(n-1) + f(n)$$

➪ Decrease-by-constant factor

$$T(n) = T(n/b) + f(n) \quad b > 1, n = b^k, k = 0, 1, 2, \cdots$$

➪ General divide-conquer

Exercise
Write the recurrence relation that describes the *Fibonacci* sequence.

$$T(n) = aT(n/b) + f(n) \quad a \geq 1, b \geq 2$$

Master Theorem

Not quite the general form (note condition on *f*).

No need to solve, sometimes

If $f(n) \in \Theta(n^d)$ with $d \geq 0$ in recurrence

$$T(n) = aT(n/b) + f(n), a \geq 1, b > 1$$

Quiz
Use theorem to determine order of growth for *a* = *b* = 2 and *d*=1. Write the divide-conquer recurrence.

$$T(n) \in \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \log n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

Solving A Recurrence

Quiz
What is the rationale (basis or reasoning) behind this approach?
Hint: not lazy!

➪ Recognize recurrence?

Lookup solution or growth results *first*

➪ Strategy to solve?

See *WolframAlpha* search box, check links in the learning resources page of course website for examples.

- ✎ Use Maple? or Wolfram MathWorld
- ✎ Backward substitutions (next)

➪ Examples

Solving A Recurrence
Solution Methods

⇨ Forward substitutions

Use relation with the generic term to generate a few terms of a summation (**series**) starting from position *n*.

⇨ Backward substitutions

$$x(n) = x(n-1) + n \text{ for } n > 0, \quad x(0) = 0$$

$$x(n) = x(n/2) + n \text{ for } n > 1, \quad x(1) = 1$$

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Challenge Exercise

- Inductive thinking in action
- Guessing a result

  Modern tools reduce the cost significantly.
- Proof by induction
- Optional bonus