Transform-Conquer
Review & Examples

Exercise
Give at least 5 examples of each type of transformation. Identify the transformation for each.

- **Instance simplification**
- **Representation change**
- **Problem reduction**

  Change to instance of another related problem you know how to solve

- **Compare**

Essentially, switch inputs, or problem or their representation. ***Ans. last slide.***

Transform-Conquer
Synthetic Division

Quiz
What's the remainder of division in this case?

⇨ Original problem, *P*

$$\begin{array}{r|l} 2x^4 - x^3 + 3x^2 + x + 5 & \\ \hline x - 3 & x_0 \end{array} \quad = 2x^3 + 5x^2 + 18x + 55$$

Coefficients of the resulting polynomial may be obtained without performing any division.

⇨ What's the reduced problem?

$$p(x)\Big|_{x=3} = 2x^4 - x^3 + 3x^2 + x - 5$$

Transform-Conquer
Problem Reduction

$$P \leq_{\mathrm{m}} Q$$

Id original problem P and its question, **reduced problem** Q and its equivalent question.

⇨ A thinking map

Transforming a geometric question to an algebraic one is a very common reduction pattern (next).

⇨ Common reduction patterns

⇨ Examples (next)

📖 **Chapter 11**

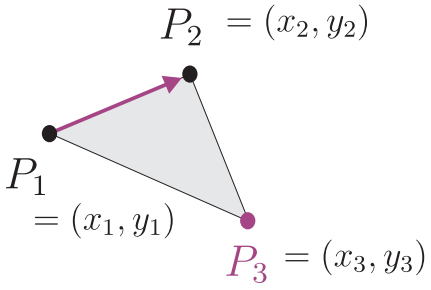
In simple terms, a reduction involves a function which maps (transforms) an instance of P to an instance in Q for all instances (i.e., get same result from either).

⇨ Major theoretical importance

Problem Reduction
Relative Point Position

➪ Equivalent question

Quiz
What's the original problem/ question? What's the reduced problem and its equivalent question? ***Ans. last slide.***



Analytic geometry is a rich resource to answer geometric questions in algorithms.

Use result from $\Delta P_1 P_2 P_3$

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = x_1y_2 + x_3y_1 + x_2y_3 - x_3y_2 - x_2y_1 - x_1y_3$$

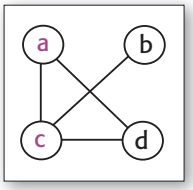
Count Paths in Graph

Quiz
What's the reduced problem and its equivalent question?

Paths of certain length

How many ways to get from a to c in *k* steps?

Exercise
How many paths of length 2 from c to c? How many of length 3 from a to c? ***Hint: get 3rd power from Wolfram Alpha, verify in graph.***



A

	a	b	c	d
a	0	0	1	1
b	0	0	1	0
c	1	1	0	1
d	1	0	1	0

A^2

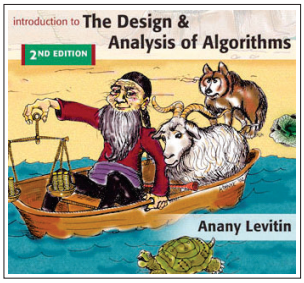
	a	b	c	d
a	2	1	1	1
b	1	1	0	1
c	1	0	3	1
d	1	1	1	2

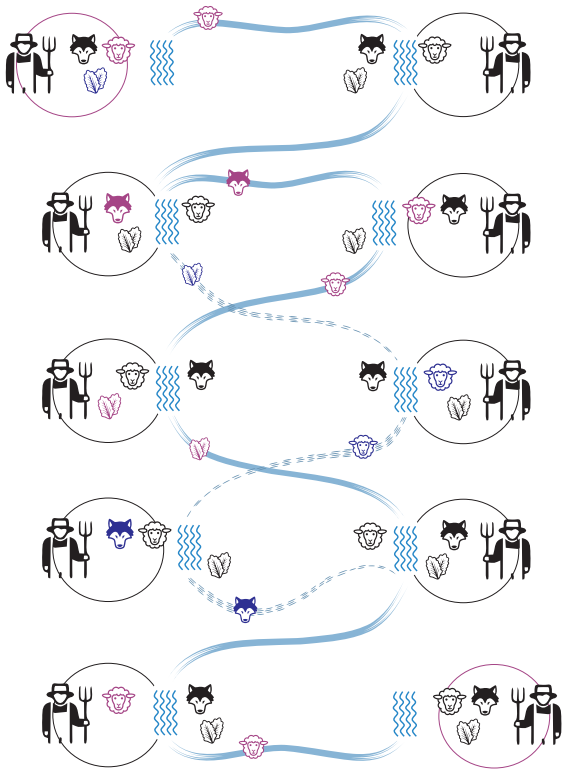
$\ldots A^k$

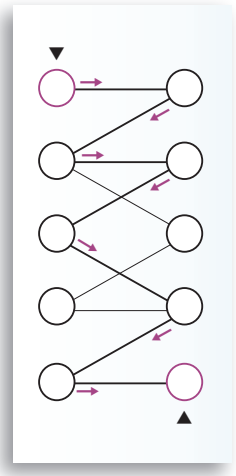
$\ldots$

Problem Reduction
Solve Puzzles

➪ State-space graph







Legal moves leading to valid positions suggest edges and verts in a graph (should it be directed?).

Quiz
What is an equivalent question in the (reduced) graph problem? Which graph algorithm can answer the question?

Quiz
Give examples from previous chapters for a reduction to a graph problem. **Hint**: review graph problems. *Ans. last slide.*

Problem Reduction
Least Common Multiple

➪ Prime factors

➪ Pen-paper procedure

Exercise
Use elementary school pen-paper procedure to compute *LCM*(60,24).

➪ Algorithm: reduce to GCD

Quiz
Determine: question of original problem, the reduced problem, reduced question, and transformation procedure.

Algorithm *Euclid*
Input Integers $m \geq 0$, $n > 0$
Output $\gcd(m, n)$ the greatest common divisor

1: **while** $n \neq 0$ **do**
2: $\quad r \leftarrow m \bmod n$
3: $\quad m \leftarrow n$
4: $\quad n \leftarrow r$
5: **return** m

Exercise
Verify the formula which calculates the *LCM* (m,n) from *GCD* (m,n)? **Hint**: do Exercise 6.6 (1).

Problem 2, Exercise 6.6

Instance simplification: switch to another instance. Representation change: switch to another representation of instance (e.g., switch data structure), or switch to alternate computation based on another way to represent the problem. In both cases, the same problem, whereas in reduction switch to another problem.

Original: position of a point relative to line from p1 to p2, question: which side of line?
Reduced: compute a determinant, equiv question: what is the sign of the determinant?

PERT (lookup term) reduced to a topological sorting in a DAG. A shortest tour of cities reduced to a Hamiltonian circuit in a complete weighted graph with min sum of weights.