

Transform-and-Conquer Representation Change

Quiz

What is the advantage of changing the representation in each case?

Rearrange to take advantage of efficient ops on new representation

Examples

 List to BST (or AVL)

 BST to 2-3 tree

 **Heap**

A heap is a simpler partially ordered $O(\log n)$ **dictionary data structure**.

The Heap

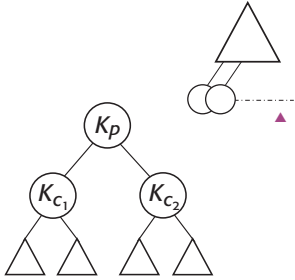
⇒ [Essentially] complete

⇒ Parental dominance

A heap is recursive, every subtree is also a heap.

Definition

- ✎ Is a binary tree
- ✎ Tree shape requirement
- ✎ Heap (key) condition

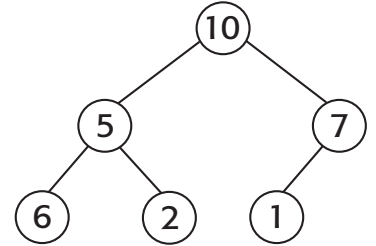
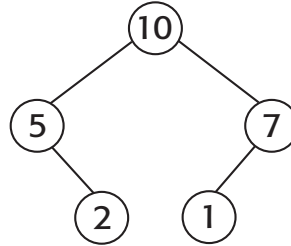
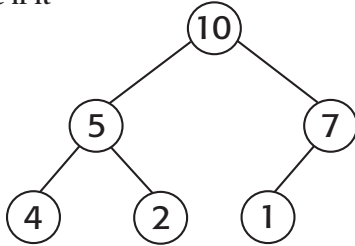



Quiz
Write the heap condition
for the keys above.

Heap Definition Which One is a Heap?

⇒ Parental node

Recall, definition-based questions are really about set membership, check criteria in definition to decide if it applies.




Quiz 
Which one is also a **full** binary tree? Does a heap need to be full? Does it have to be perfectly balanced? give an example. Can a search tree be both BST and a heap?

Heap Definition

More About Heaps

⇒ **Properties**

-  Root has largest (maximum) key
-  Height of n -node heap: $h = \lfloor \log_2 n \rfloor$
-   More ...

⇒ **Applications**

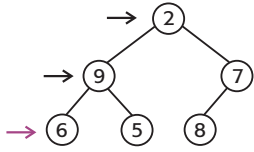
Characteristic operations
efficient if keys stored as a
heap instead of a list.

-  Priority queues
-  Heapsort: efficient in-place sort

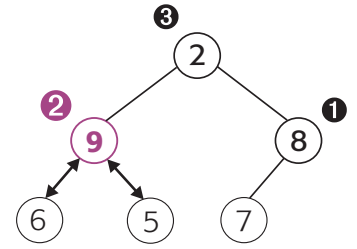
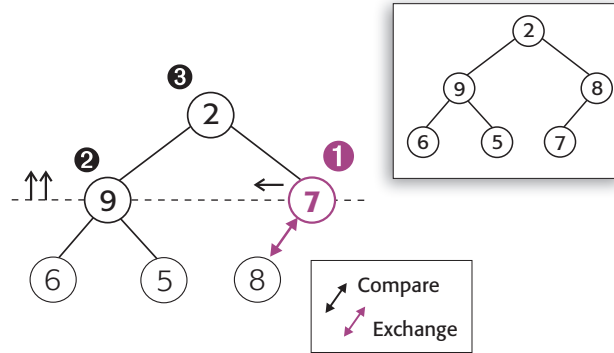
Heap Construction

⇨ Bottom-up heapify

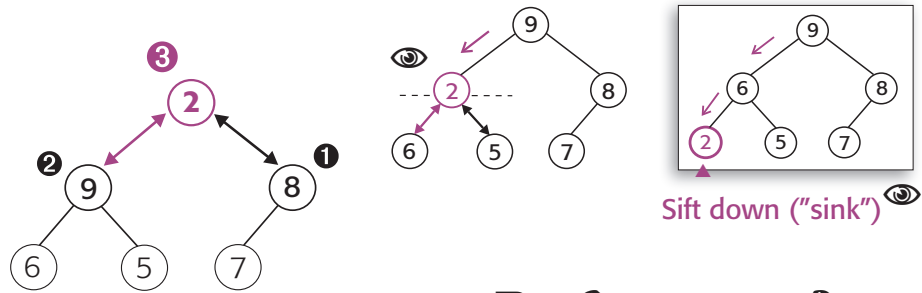
2,9,7,6,5,8



Big tree shows major iteration (labeled 1-3), small (boxed) shows result after heapify.



For each major iteration potentially, while not heap repeatedly keep sifting down to fix.

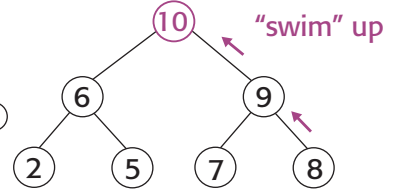
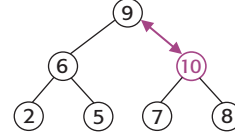
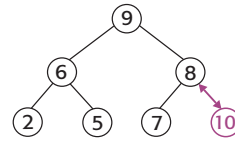
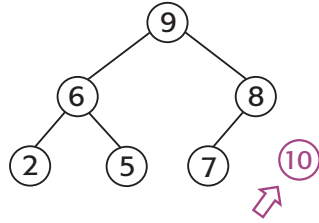


Performance?

Heap Operations

Key insert

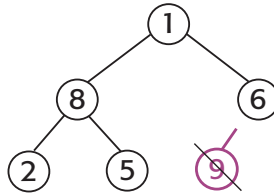
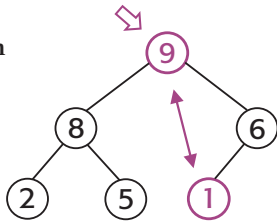
Quiz
 What are the efficiencies of these operations?



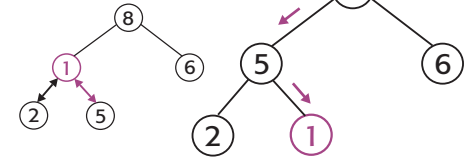
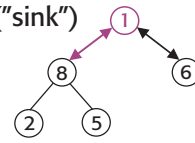
Exercise
 Write pseudocode for pen-paper application to describe operations (note where iteration may, potentially, be needed).

Max key delete

Eye
 To check or dequeue highest priority item in a heap-based priority queue is very efficient.



Eye Sift down ("sink")



Array Representation

Quiz

What are the indices of children of node i ? What's the index of the last **parental** node?

An array representation is attractive because of the simple relations between node indices.

Exercise

Write the array after each change in the heap construction example?

