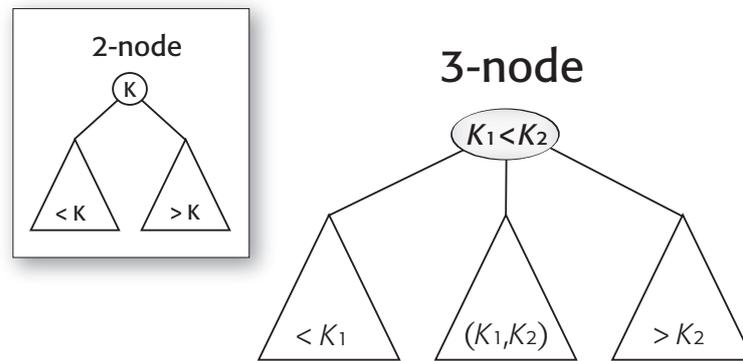


# Balanced Search Trees

## 2-3 Trees

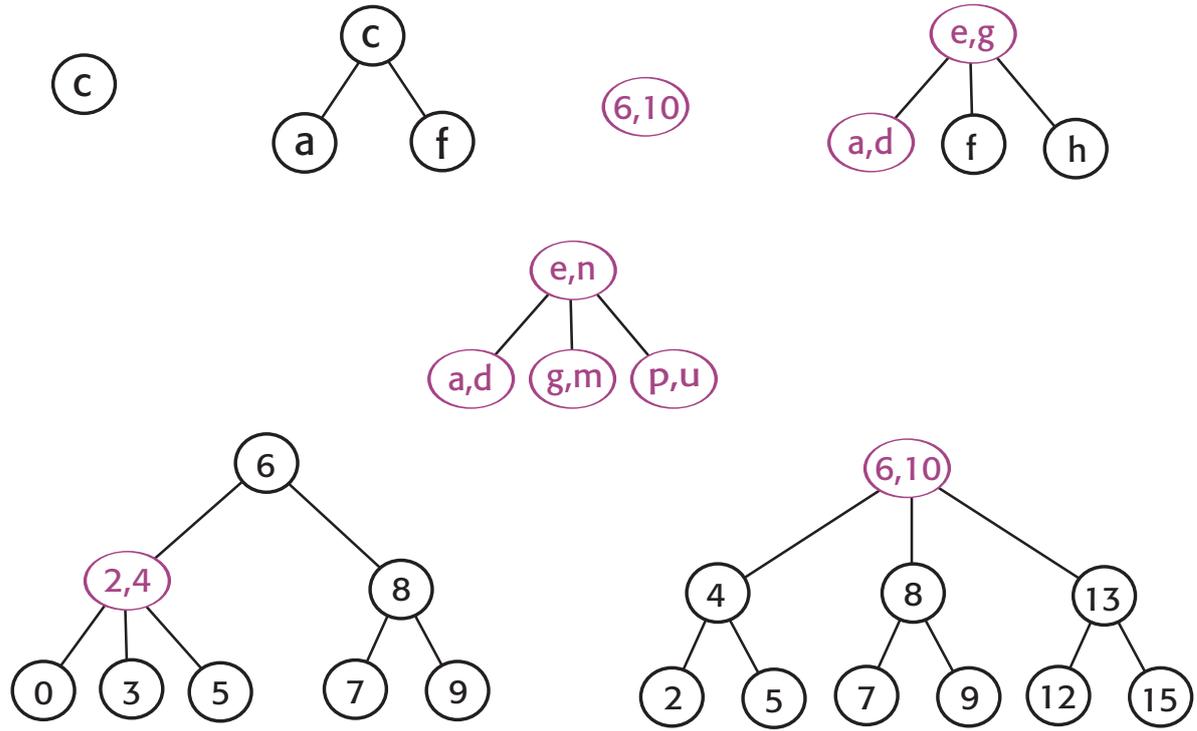
⇒ Representation change



⇒ **All leaves on same level**

⇒ **Insert new key in a leaf**

# 2-3 Trees Examples



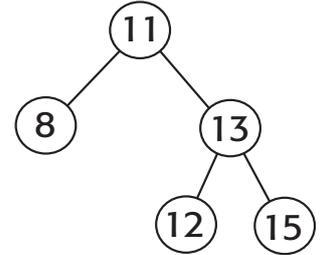
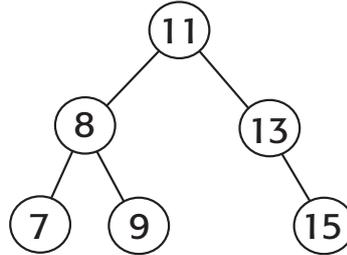
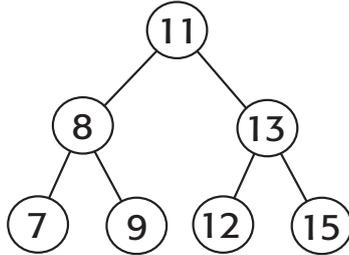
# 2-3 Trees Definition Exercise

- ⇒ Full binary tree
- ⇒ Complete tree

Not every valid BST is a 2-3 tree. (Why?)

## Which BST is also a valid 2-3 tree?

Definition-based questions are really about set membership. Check membership criteria in the definition to decide if it applies. (Note b is tricky).



### Quiz

Is tree (c) an AVL? (Determine node balance factors).

### Exercise

Show in a Venn diagram how BST, AVL, 2-3 trees relate.

# Building 2-3 Trees

⇒ **Empty: insert as root**

⇒ **Not empty**

 Find suitable leaf via key condition

 Insert key in leaf (next)

# Building 2-3 Trees

## Insert Key Procedure

⇨ Key promotion

😊 ⇨ **If leaf 2-node insert as 2nd key**

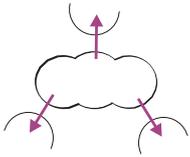
⇨ **If leaf 3-node**

✎ Split to two 2-nodes: left, right

✎ Insert smallest key left, largest right

✎ Promote middle to parent

✎ If parent empty make middle new root  
else insert middle in parent



Basic insert procedure  
may repeat to restore 2-3  
conditions.

# Building 2-3 Trees Example

2,8,3,7,5,6,4

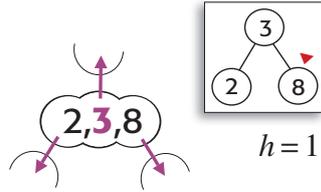
2



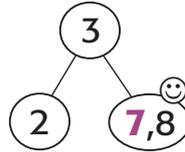
8



3

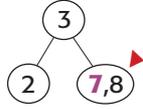


7

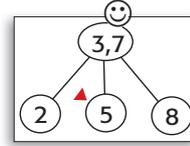
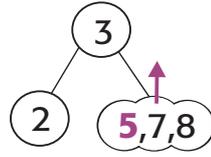


# Building 2-3 Trees Example (Cont.)

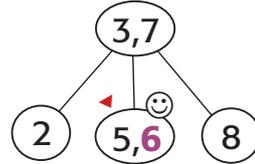
2,8,3,7,5,6,4



5

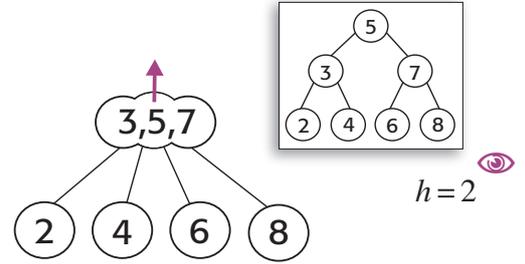
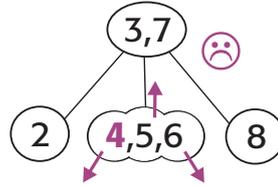


6



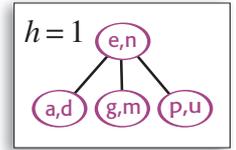
**Quiz**  
 How many times *insert* is called in each case?

4



# 2-3 Trees Conclusions

## 7.4 $\Rightarrow$ Importance



### Quiz

In example, what is the shortest sequence of keys leading to a height of 2? What's the longest sequence? **Hint:** see final tree in previous slide and tree in this slide (fig).

## $\Rightarrow$ Dictionary operations (worst)

## $\Rightarrow$ Build efficiency

### Exercise

In terms of practicality (implementation costs), efficiency of build and dictionary ops.

## $\Rightarrow$ Compare with BST, AVL