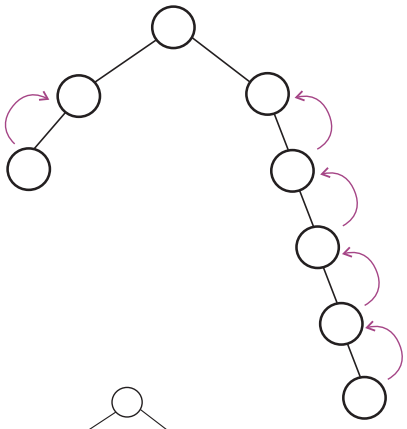
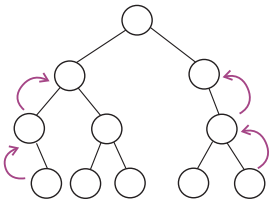
Transform-Conquer
Balanced Search Trees

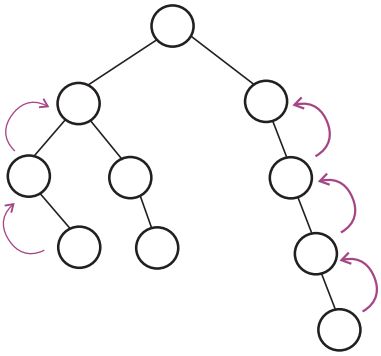
- 🔑 ➪ **Dictionary**
- ➪ **Node balance**
- ➪ **Perfect balance**

Exercise
Define the term **dictionary** data structure, give examples. **Hint**: check 🕮1.4.

Quiz
What is the efficiency of dictionary ops in a binary search tree (BST)?







$n = ?$

👁

Quiz
A perfectly balanced tree leads to optimal dictionary ops due to its optimal height, however, is perfect balance required for optimal height?.

Balanced Search Trees
AVL Trees

- Balance factor
- AVL balance

Like sorting a list to gain a $O(\log n)$ search, can we improve the $O(n)$ **dictionary operations** of general binary search tree?

➪ Motivation

Quiz
Define the **balance factor**. Which tree from previous slide could be an AVL?

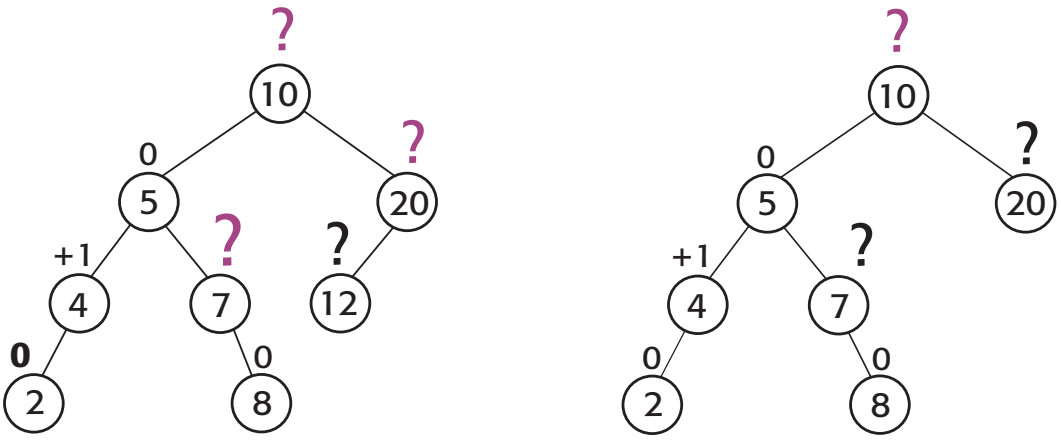
➪ Definition

➪ Performance promise

Dictionary operations must maintain <u>both</u> BST structure <u>and</u> AVL balance.

➪ Extra operations: rotations

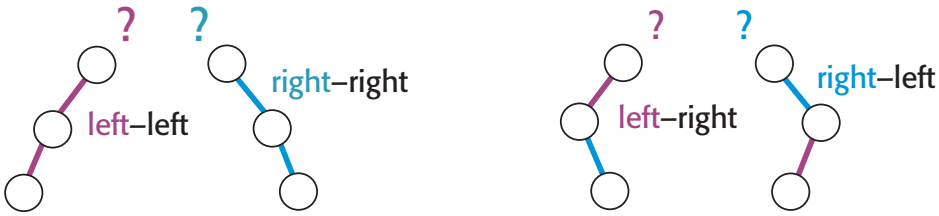
AVL Tree Definition
Balance Factor Exercise



Which BST is also a valid AVL tree?

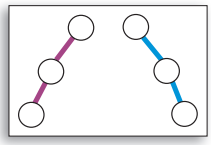
AVL Tree Operations
Observations

Exercise
Determine indicated balance factors.



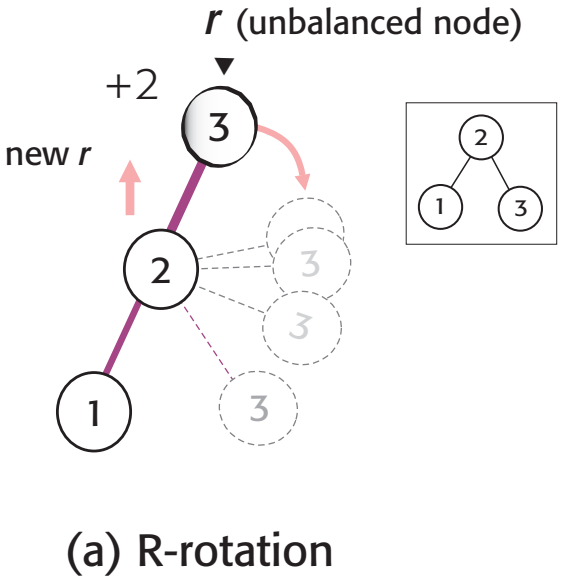
- ⇨ **Insert/delete of a key in an AVL causes max imbalance of ±2**
- ⇨ **Imbalance patterns**
- ⇨ **Correction**

AVL Tree Operations
Simple Rotations

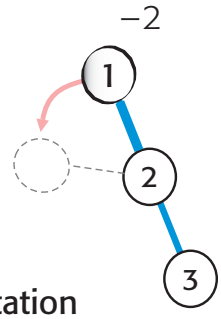


Both unbalanced node and its subtree on same side.

Left-left child pattern leading to single right rotation to restore balance.



(a) R-rotation



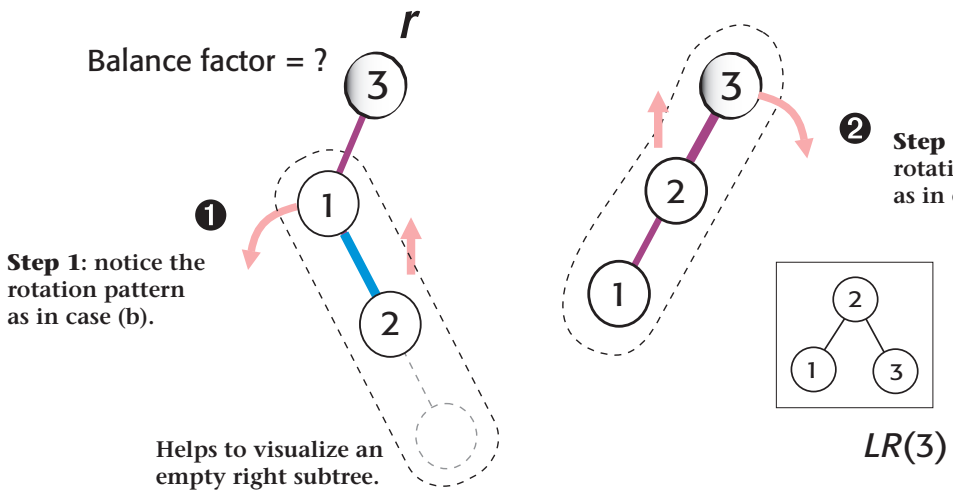
(b) L-rotation

Quiz
Where does the term **rotation** come from?

AVL Tree Operations
Double Rotations

Step 1: L-rotate subtree of *r*
Step 2: R-rotate *r*

Left-right child pattern leading to **LR**-rotation.



Exercise
Repeat for case (d), RL double rotation.

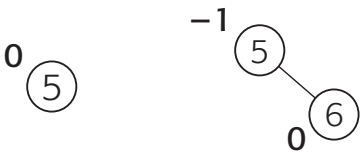
(c) LR-rotation

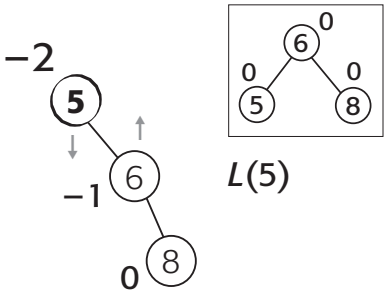
Building AVL Tree Example

AVL insert procedure must always update node balance factors, and perform rotations as needed.

5, 6, 8, 3, 2, 4, 7

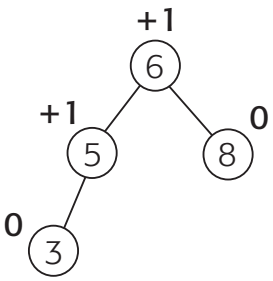
Quiz
Suggest a formula for updating the balance factor.

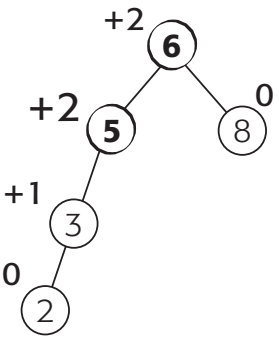


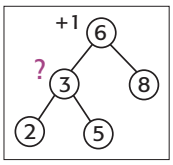


$L(5)$

Quiz
What if more than one node breaks the AVL balance?



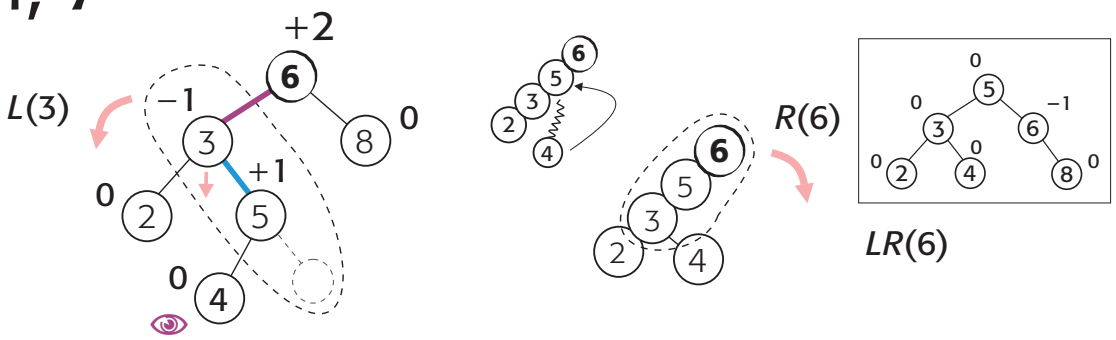




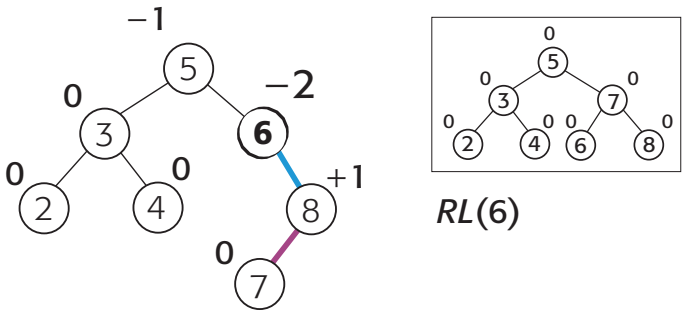
$R(5)$

Building AVL Tree Example

5,6,8,3,2, 4, 7



Complication (3 rotates left of 5) leading to 2 left subtrees (?)



Quiz
In what ways would an algorithm to insert in AVL tree be different from one that inserts in a BST?

Transform-Conquer: AVL Tree
Summary

An AVL tree actively maintains balance for optimal or close to optimal dictionary operations.

➪ **Where's the transformation?**

➪ **Performance gains**

➪ **Interesting average**

➪ **Main disadvantage**