Convex Hull Review

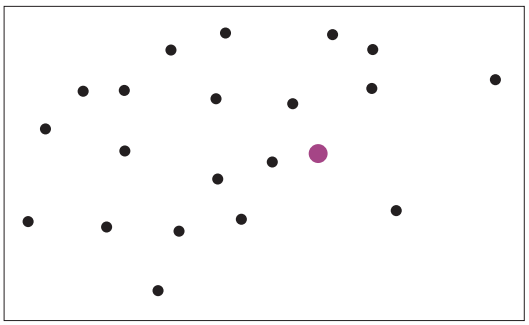
➪ Extreme point

Quiz
Is the set of points $P_1 \ldots P_n$ a **convex set**?

Quiz
Is the highlighted point in the **convex hull** of the set? Can it be a vertex of an enclosing polygon? Why?

Exercise
Draw the **convex hull** of the set.

$$P_1 = (x_1, y_1), \ldots, P_n = (x_n, y_n)$$



Examine line segments (distinct pairs of points) against each of the remaining n-2 points.

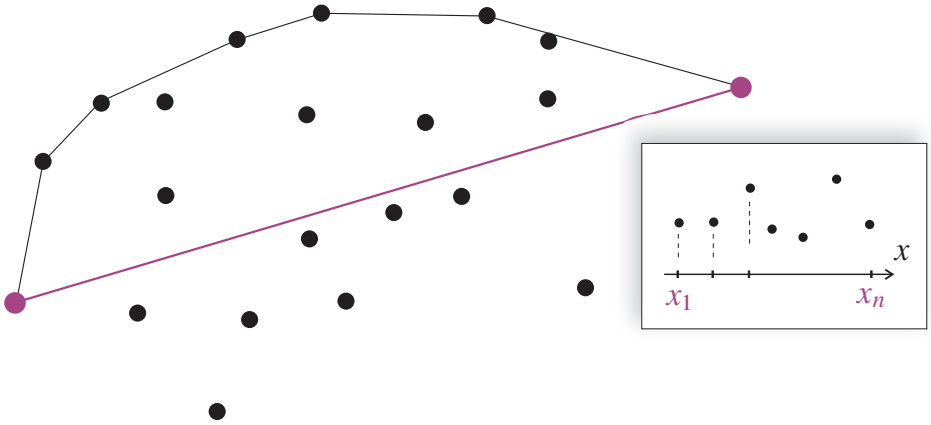
Brute force $O(n^3)$, can we do better?

Divide-Conquer Convex Hull
Initializations

➪ Upper/lower hull

Quiz
How to quickly determine left-most and right-most points? What about points on $\overline{P_1 P_n}$?

$$P_1 = (x_1, y_1), \ldots, P_n = (x_n, y_n)$$

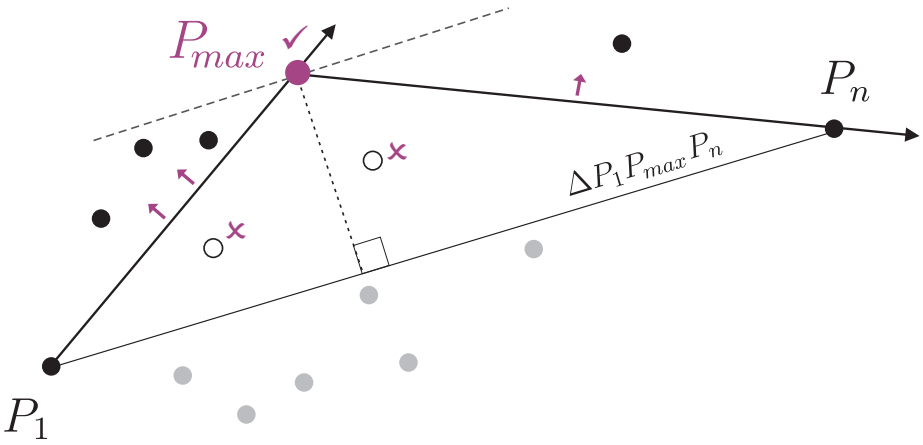


Basic Procedure

3 Observations

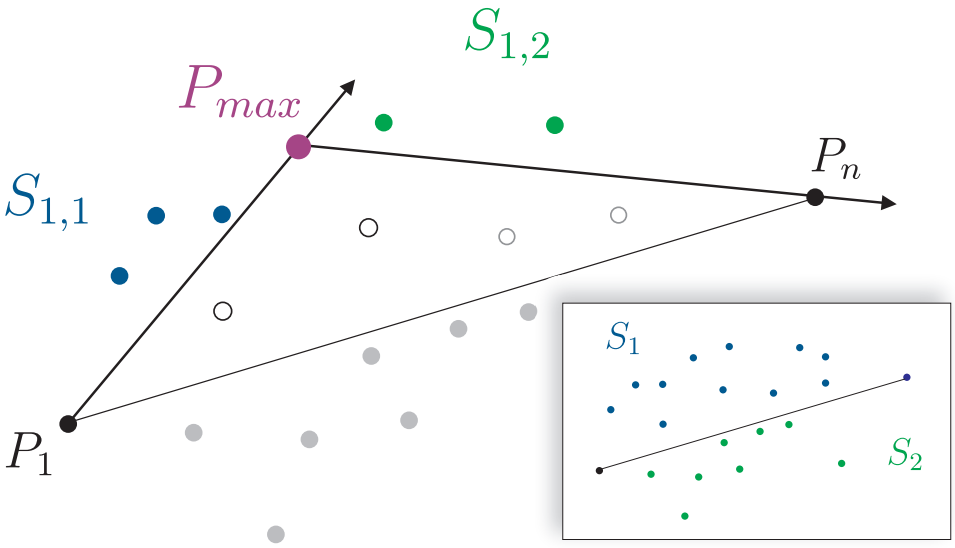
$P_{\max}$ divides the upper set into 3 sets.

Quiz
How can the highlighted point be selected? **Hint**: see details slide.



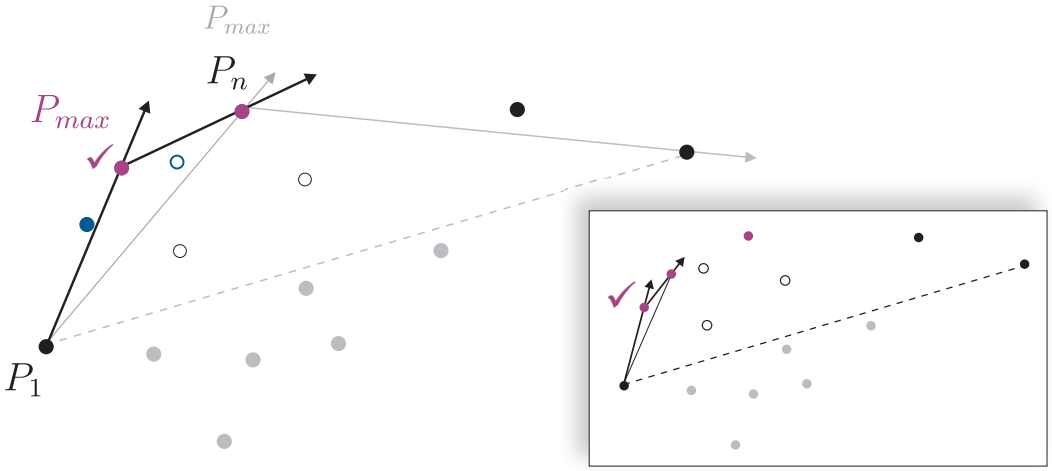
Divide-Conquer Convex Hull
A Partition

Each partition identifies a vertex on the upper hull + creates 2 sets to check next.



Divide-Conquer Convex Hull
Quickhull

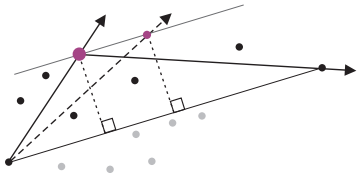
Previous P_{max} becomes P_n for this round.



⇨ Repeat recursively

🕮 ⇨ Merge procedure

A Quickhull Algorithm
Computational Details



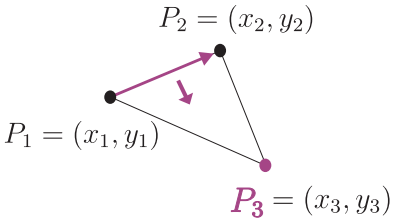
Quiz
What if many points qualify to partition set?

➪ How to determine P_{max} ?

Quiz
What is the sign of the determinant in the figure?

➪ Use result for $\Delta P_1 P_2 P_3$

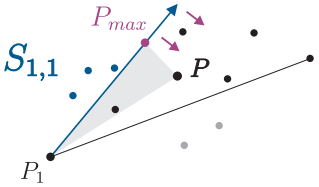
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = x_1y_2 + x_3y_1 + x_2y_3 - x_3y_2 - x_2y_1 - x_1y_3$$



Quiz
Write an expression for $S_{1,2}$.

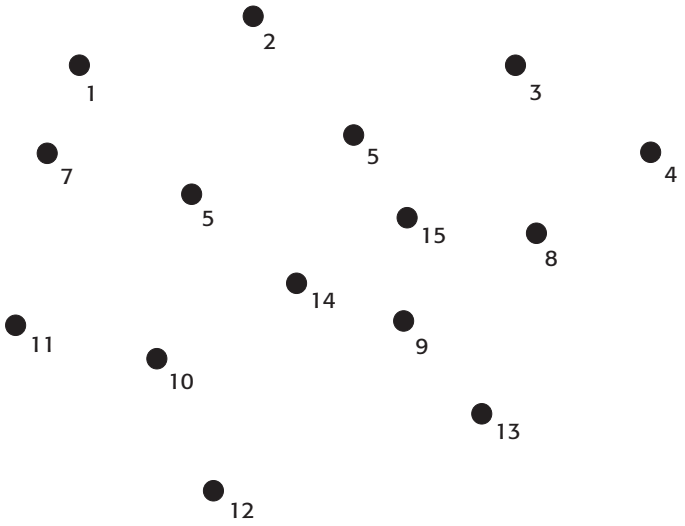
➪ To determine $S_{1,1}$

$$\forall P = (x, y) \in S_1, \text{ check } \det \Delta \overrightarrow{P_1 P_{max}}\, P$$



A Quickhull Algorithm
Exercise

Exercise
Perform a *Quickhull* by hand. Show for each step: Pmax, excluded points, sets of points to consider next, and the resulting convex hull vertices.

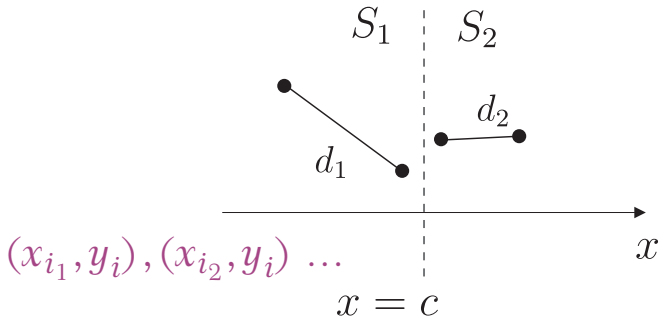


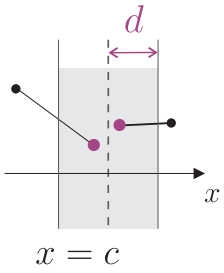
Divide-Conquer Closest Pair
Basic Procedure

$S = \{P_1, P_2, P_3, P_4\}$

$(x_1, y_1) \ldots$

Quiz
How to quickly determine a partition point C? How many points in each subset in general?
Hint: think about algorithm inputs (x_{ik}: *i* original index, *k* ordered/sorted index).



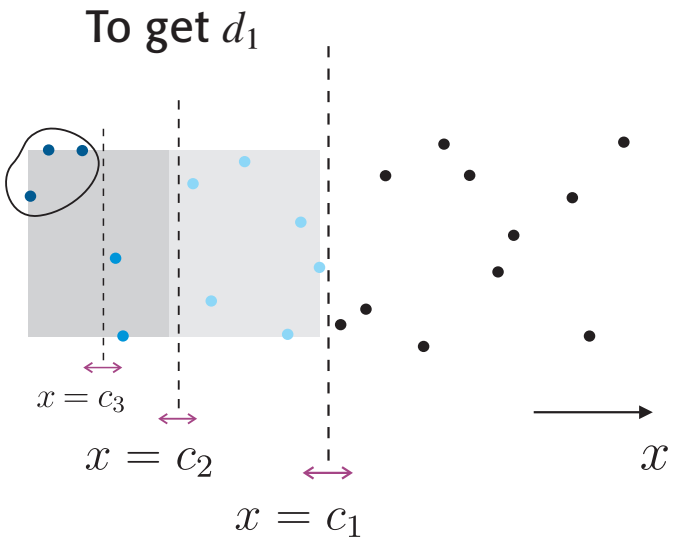


Determine separately, combine

- $d = \min(d_1, d_2)$
- Check pairs around $x = c$

Divide-Conquer Closest Pair
Repeat Recursively

Exercise
Design top level pseudocode without detailing the merge step.



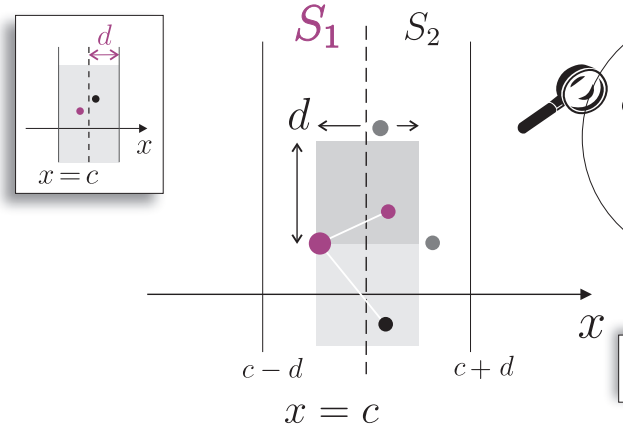
Divide if points > 2 (or 3) otherwise return a distance.

Keep dividing until set is 3 or less points

Divide-Conquer Closest Pair
Merge Procedure

Points outside clearly farther than *d* but not all inside are closer.

Even inside P_i's box some points are farther away than *d*.



Quiz
Show that the distance between the points is > *d*, if the x-corrdinate of the top point is at distance *d*/2.

$$\forall P_i \in S_1(c-d \le x \le c),\ \text{check } \{P_j \in S_2(c \le x \le c+d)\}$$

- Major result: no more than six neighbors to consider per point

Performance Results

Quiz
Compare problem size reduction in each case.

➪ **Recurrence pattern**

Quiz
Write the recurrence, determine order of growth of solution.

➪ **Divide-conquer closest pair**

➪ **Quickhull worst and average**

Exercise
Lookup best and average case efficiencies of the brute force convex hull of Chapter 3.

➪ **Compare to brute force**