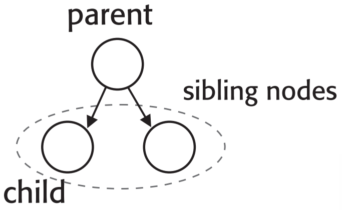
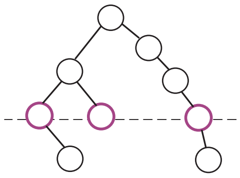
1.4 Review

Binary Trees

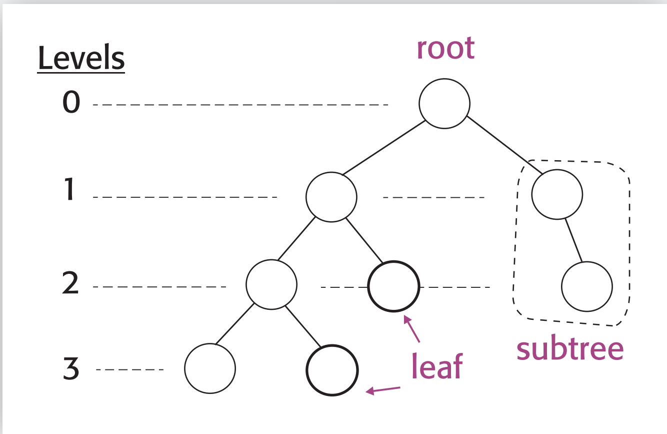
- Node depth
- Tree height



Quiz
Define a binary tree (*note recursive/self-repeating statement).*

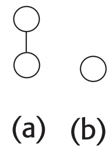


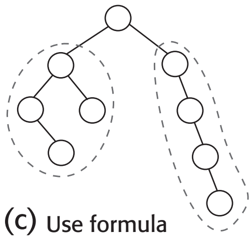
Quiz
What's the **depth** in each case? What's the maximum **level** of a n-node binary tree?



$$H = \max\{H_L, H_R\} + 1$$

Quiz
What's the **height**? What about an empty tree? Why?





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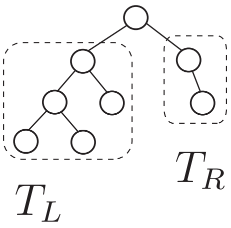
Divide-and-Conquer
Binary Tree Height



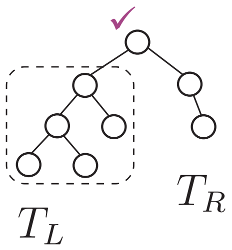
Algorithm $Height(T)$
Input A binary tree T
Output The height of T

```
if T = φ then
        return −1
else
        return max{Height(T_L), Height(T_R)} + 1
```

Algorithm $Height(T)$
Input A binary tree T
Output The height of T

```
if T = φ then
    return −1
H_L = Height(T_L)
H_R = Height(T_R)
return max{H_L, H_R} + 1
```

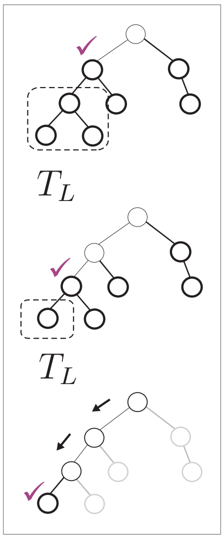
Divide-Conquer Tree Height Analysis



Algorithm $Height(T)$
Input A binary tree T
Output The height of T

```
if T = φ then
    return −1
else
    return max{Height(T_L), Height(T_R)} + 1
```

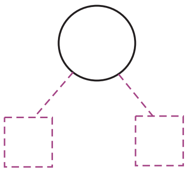
```
if T = φ then
    return −1
return max{Height(T_L), Height(T_R)} + 1
```

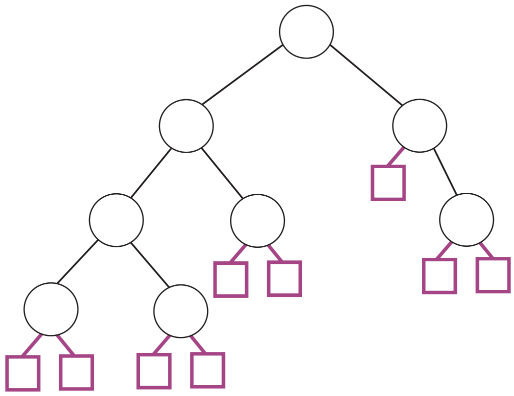


- ➪ **Problem size, examples**
- ➪ **Basic operation?**
- ➪ **A recurrence (id worst case)**

Divide-Conquer Tree Height
Efficiency: Count Nodes

Observe,



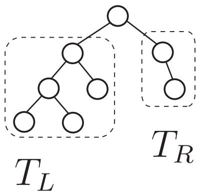


Quiz
How many **external nodes** if a tree has n nodes? What's the total number of internal and external nodes?

✎ 1 addition/internal node

✎ 1 <u>extra</u> comparison/external node

Conclusions



Algorithm *Height(T)*
Input A binary tree T
Output The height of T

```
if T = φ then
    return −1
H_L = Height(T_L)
H_R = Height(T_R)
return max{H_L, H_R} + 1
```

Exercise
Add code to monitor numbers of comps and additions, compare to formulas from textbook.

➪ Natural divide-conquer

➪ Processing pattern

Definition suggests family of recursive algorithms

- ✎ Tree traversals
- ✎ Leaf counter
- ✎ Do exercises (see practice code) Exercise 2