

# Algorithm Efficiency

## Chapter 2

⇒ **Space complexity**

⇒ **Time complexity**

⇒ **How well it performs?**

 Space efficiency

▶  Time efficiency

⇒ **A framework for analysis**



A note on homework exercises and weekly tutorial sessions.

⇒ **Approach to chapter (caution)**

# Analysis Framework

- ⇒ **Input size as parameter**
- ⇒ **Measuring run time**
- ⇒ **Run time growth**
- 👁 ⇒ **Algorithm efficiency**
- ⇒ **Types of efficiency analysis**

# Analysis of Algorithms Orders of Growth

These functions arise naturally from the way we design algorithms.

## How fast $f$ grows as $n$ increases?

**TABLE 2.1** Values (some approximate) of several functions important for analysis of algorithms

$n$	$\log_2 n$	$n$	$n \log_2 n$	$n^2$	$n^3$	$2^n$	$n!$
10	3.3	$10^1$	$3.3 \cdot 10^1$	$10^2$	$10^3$	$10^3$	$3.6 \cdot 10^6$
$10^2$	6.6	$10^2$	$6.6 \cdot 10^2$	$10^4$	$10^6$	$1.3 \cdot 10^{30}$	$9.3 \cdot 10^{157}$
▶ $10^3$	10	$10^3$	$1.0 \cdot 10^4$	$10^6$	$10^9$		
$10^4$	13	$10^4$	$1.3 \cdot 10^5$	$10^8$	$10^{12}$		
▶ $10^5$	17	$10^5$	$1.7 \cdot 10^6$	$10^{10}$	$10^{15}$		
$10^6$	20	$10^6$	$2.0 \cdot 10^7$	$10^{12}$	$10^{18}$		

**Exercise**  
Determine resulting growth when  $n$  doubles (increases twofold) for each function?

# Analysis of Algorithms

## Types of Efficiency

⇒ **Worst-case efficiency**

⇒ **Average-case efficiency**

Runtime of a *selection sort* always depends on list size only, is this true for the search?

Algorithm *SequentialSearch*

**Input** Array  $A[0..n-1]$ , search key  $K$

**Output** Index of first element of  $A$  to match  $K$ , otherwise  $-1$

1:  $i \leftarrow 0$

2: **while**  $i < n$  and  $A[i] \neq K$  **do**

3:      $i \leftarrow i + 1$

4: **if**  $i < n$  **then return**  $i$    ▷ found if  $i$  in range

5: **return**  $-1$



Runtime growth can vary for the same input size.

# Analysis of Algorithms A General Plan

🕒 ⇨ Amortized efficiency

- ① Select suitable input size parameter
- ② Identify suitable basic operation
- ③ Check dependence of basic op
- ④ Determine count  $C(n)$ , somehow
- ⑤ Determine order of growth of  $C(n)$

↪ Some algorithms have different counts for the same input size.

↪ Run time grows similar to growth of basic operation count.