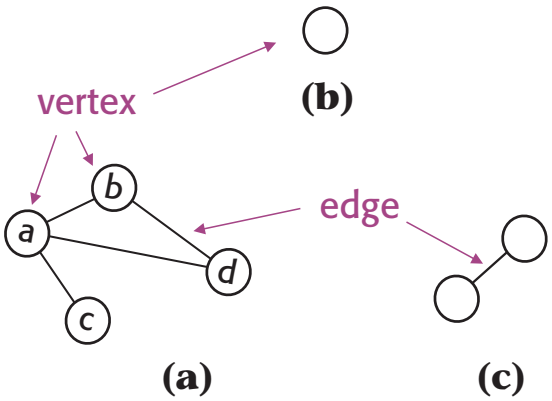
Graphs: The Basics

Defs & Terms

16 key-terms

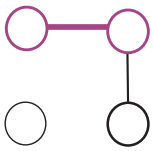
- [1] **Graph,** [2] **vertex,** [3] **edge**
- [4] **Adjacent vertex**
- **Vertex degree** [5]



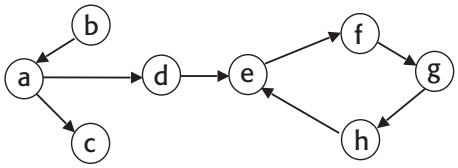
(a) (b) (c)

$$G = \langle V, E \rangle$$
$$V = \{v_i (1 \le i \le n)\}$$
$$E = \{(u, v) | u, v \in V\}$$

Exercise
Write a formal (math) definition to describe vertex adjacency.



(d)

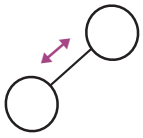


(e)

Graphs: The Basics
Edge Properties

- [6] **Undirected graph**
- **Digraph** [7]
- [8] **Complete, Dense/sparse** [9]

Exercise
Specify graphs (a) and (e) using the definitions.



Edge direction

Undirected graph

$$\overset{\text{iff}}{\iff} \forall (u,v) \in E, (u,v) \equiv (v,u)$$

Directed graph: def, example

Quiz
What's the efficiency of any algorithm that traverses all edges of undirected graph?

Number of edges (undirected)

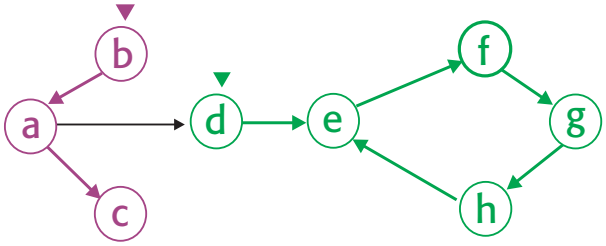
Graphs: The Basics

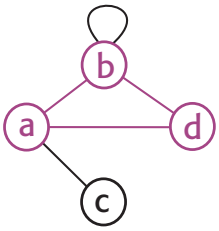
More Defs

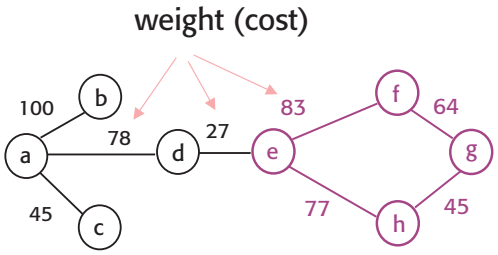
- [10] Path, cycle/loop[11]
- Path length[12]
- [13] Weighted graph

Exercise
Specify the highlighted paths. Identify simple paths giving a reason if not? What's the length in each case?

Quiz
Is a Hamiltonian circuit a **simple path**? Why?





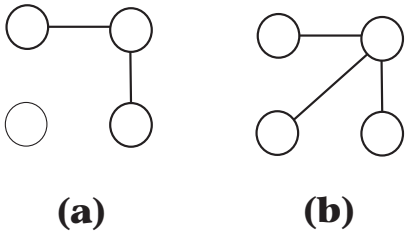


Graphs: The Basics

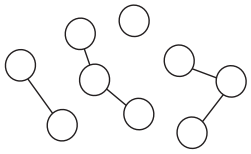
Main Properties

- Connected graph [14]
- Connected component [15]
- Acyclic graph [16]

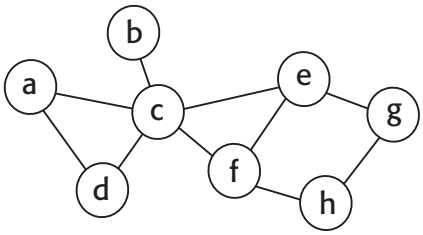
Exercise
How many connected components?

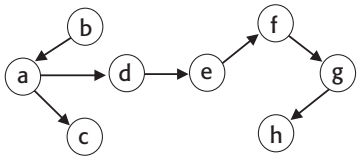


(a) **(b)**



Exercise
List as many cycles as you can. How many are unique cycles? (Minimal? or perhaps some distinct edges? Look it up.)

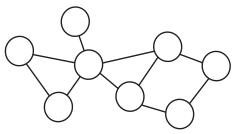




Directed acyclic graph (*DAG*)

Motivation

➪ Interesting graph problems



- ✎ Find unreachable vertices
- ✎ Find an articulation point
- ✎ Find a shortest path

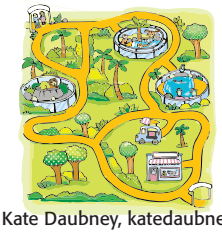
Exercise
Find all **articulation points** (definition in textbook). Hint: check each vert in turn.

➪ A brute force approach

Check all vertices and all/most edges

Graph Traversals

➪ Traversal tree



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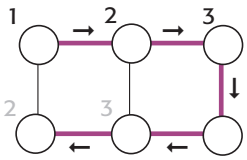
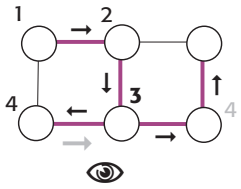
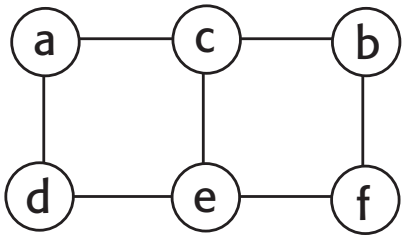
Explore vertices and edges

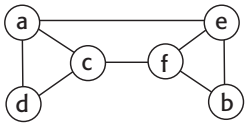
A natural approach to visiting all verts once is to explore the most recently discovered paths leading to unseen verts.



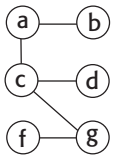




(a)



(b)



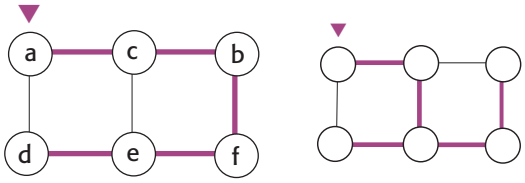
(c)

🕮 Convention

Graph Traversals
Depth First Search

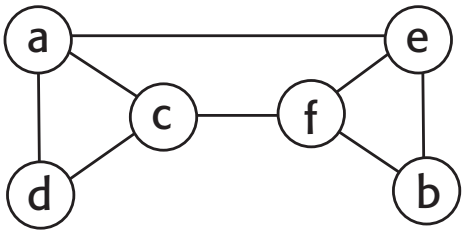
- Tree edge
- Back edge
- Dead-end vertex

Exercise
Draw the **traversal tree** and show unused edges (dotted), observe its properties.



(a)

Seems natural to **backtrack**, if needed, along last explored edge to ensure visiting a most recently discovered new path via last seen vert.



(b)

Exercise
Perform a DFS to determine the number of cycles, compare to previous answer.