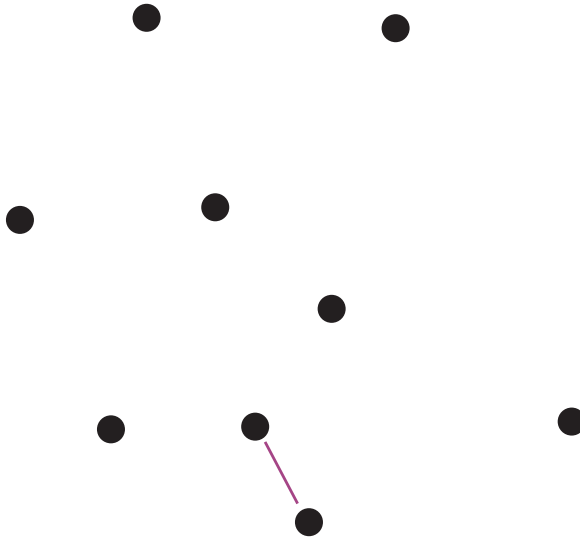



# Closest Pair of Points



**Quiz**  
Describe a **brute-force approach** for finding the closest pair of points.



# Closest Pair of Points Brute Force Algorithm

 **Quiz**  
Compare this brute force solution to selection sort?

**Quiz**  
What exactly is the output?  
(Give examples.)

**Algorithm** *BruteForceClosestPair*


**Input** Set of  $n \geq 2$  points  $P_1 = (x_1, y_1), \dots, P_n = (x_n, y_n)$

**Output** Indices of 2 points with smallest distance in set

1:  $dmin \leftarrow \infty$

2: **for**  $i \leftarrow 1$  **to**  $n - 1$  **do**

3:     **for**  $j \leftarrow i + 1$  **to**  $n$  **do**


4:          $d \leftarrow \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$  

5:         **if**  $d < dmin$  **then**

6:              $dmin \leftarrow d; index1 \leftarrow i; index2 \leftarrow j$

7: **return**  $index1, index2$

1	2	3	4
1	x	x	x
2	x	x	x
3	x	x	x
4	x	x	x

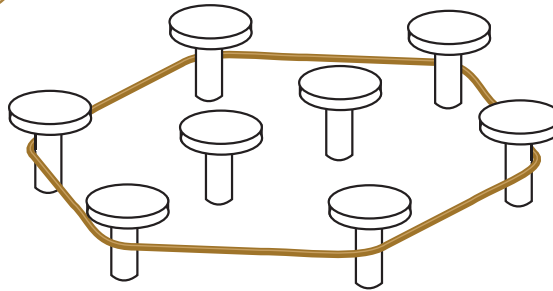
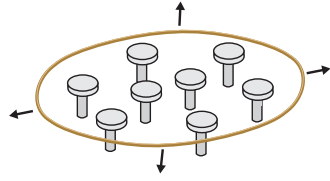
 Note loop indices generated similar to selection sort.

**Quiz**  
How can the run-time be improved? **Hint:** study the pseudocode.

## Performance?

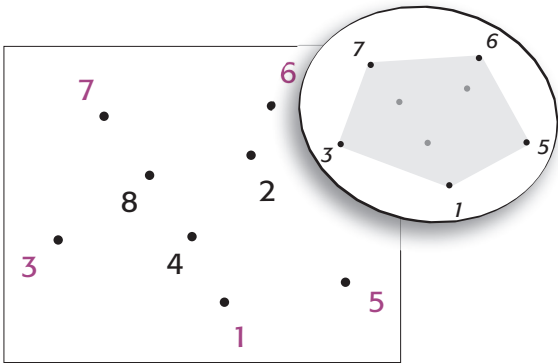
# Convex Hull Problem

⇒ Computational geometry



## Quiz

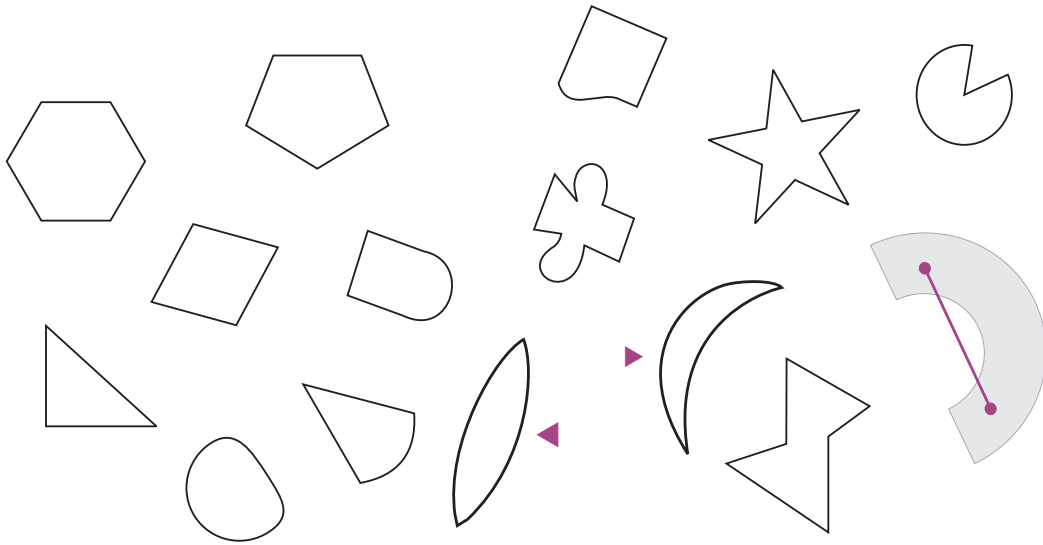
What can generally be observed about the enclosing shape in each example?



<https://learnopencv.com>

# Convex Set of Points

⇒ Convex set

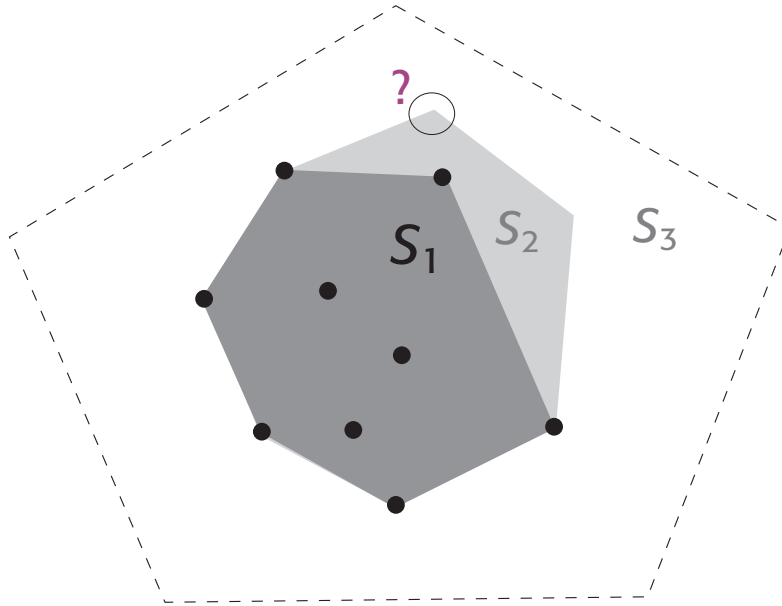


**Quiz**  
Are points  $P_1 \dots P_8$  from  
previous slide a convex set?



# Convex Set of Points The Convex Hull

- ⇒ Convex polygon
- ⇒ Convex hull



**Quiz**  
Which of the convex sets  $S_1, S_2, S_3$  is the **convex hull** for the shown set of points?

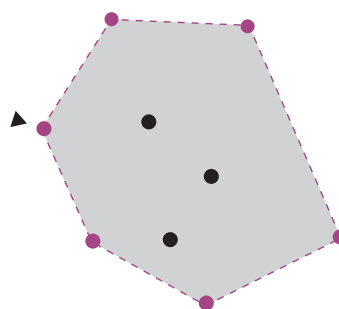
# The Convex Hull Theorem

⇒ Polygon vertices

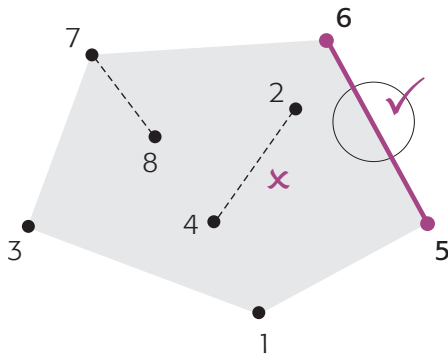


The convex hull of any set  $S$  of  $n > 2$  points (not all on the same line) is a *convex polygon* with the **vertices at some of the points of  $S$** .

Note  $S_2$  (previous slide) is convex but some of its vertices are not in  $S$ .



# Brute Force Convex Hull Algorithm Idea



- ⇒ Extreme point
- ⇒ Line segment

**Exercise**  
Write the input and output parts of a pseudocode.

- ⇒ Observation: line segments
- ⇒ Brute force approach
- ⇒ Steps outline, efficiency
- ⇒ Line segment test (later)



May be performed without a formula in a pen-paper application.

# Brute Force Convex Hull Programming Challenge

## 📖 Exercise 3.3-11

**Hint:** use pseudocode from *closestPair* as starting point.

⇒ **Write proper pseudocode**

⇒ **Code your solution**

You can visually see if your solution is correct.

⇒ **Use JSDraw to draw c. hull**

There is a code demo in the assignment support page to help with the code.

⇒ **Test instance (homepage)**



[hashimi.ws/cs223/assignment.php](http://hashimi.ws/cs223/assignment.php)



# Brute Force Convex Hull Line Segment Test



**Exercise**  
Write a pseudocode for the brute force algorithm described in the lecture. **Hint:** use the closest pair pseudocode as starting point.

